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Discovering Colorons at the Early Stage LHC

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Abstract

We investigate the prospects for the discovery of massive hyper-gluons using data from the early runs of the CERN Large Hadron Collider with $\sqrt{s} = 7$ TeV and assuming an integrated luminosity of 1 fb^{-1}. A phenomenological Lagrangian is adopted to evaluate the cross section of a pair of colored vector bosons (coloron, $\tilde{\rho}$) decaying into four colored scalar resonances (hyper-pion, $\tilde{\pi}$), which then decay into eight gluons. We include the dominant physics background from the production of $8g$, $7g1q$, $6g2q$, and $5g3q$. We find an abundance of signal events and that realistic cuts reduce the background enough to establish a 5σ signal for $m_{\tilde{\pi}} \lesssim 220$ GeV or $m_{\tilde{\rho}} \lesssim 733$ GeV.

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Introduction.– With the LHC beginning to accumulate data, we look forward to a new era of high-energy physics as we explore multi-TeV energy scales. In addition to the search for the Higgs boson as a completion of the Standard Model, many scenarios have been considered for the discovery of new physics. Often, the discovery potential provided by the LHC's unprecedented collision energies is mitigated by the prevalence of jets derived from Standard Model processes. New physics which proceeds through weak interactions, such as Higgs production, must be carefully separated from large, strong-force produced backgrounds via judicious selection cuts when jets are involved.

It is also possible that new physics will manifest itself through the strong force. If new colored particles exist at TeV scales, they will be discovered through decays into jets. One generic possibility is a massive vector boson in the color-octet representation [1–3]. Such a particle has been dubbed a coloron. According to this scenario several theories of physics beyond the Standard Model give rise to colorons such as topcolor model [1] and Kaluza-Klein excitations of the gluon in the universal extra-dimensional models [4, 5]. In Ref. [6], Kilic, Sundrum and Okui showed how a coloron, as well as a scalar octet, can emerge as the low energy states of an effective theory arising from a simple model of new, strongly interacting fermions charged under this new strong interaction as well as QCD color. In this letter we follow their analysis closely, as well as the subsequent treatment found in Ref. [7].

Briefly, if we suppose that there exists a new, strong force, termed hypercolor, it may become confining at higher energies compared to the strong QCD force. Fermions which carry hypercolor will form bound states which are hypercolor singlets but which may carry QCD color quantum numbers. In particular, if these "hyperquarks" are also triplets of QCD then their lightest bound states will be color octets. These are like the ρ meson octet of ordinary QCD, but are hypercolor singlet and color octet. Analogous to the breaking of chiral symmetry in the standard model, this model will produce relatively light hyper-pions as pseudo-Goldstone bosons in the octet representation.

These colorons, and scalar octets can have an interesting phenomenology. Naively one might think that light octets are severely constrained by dijet searches at the Tevatron. However, in this model the hyper-pions couple sufficiently weakly to gluons to leave an interesting parameter range and the hyper-rhos have only a small branching fraction to decay into two quarks or two gluons. Rather, the hyper-rho decays predominantly to hyper-pions, each of which then decays to a gluon pair. Thus the dominant signal for resonant production of each hyper-rho is a 4-jet decay chain.

On the other hand, there are several processes which pair produce hyper-pions without a resonant hyper-rho. Combined with the loss of jet resolution during showering, hadronization, and reconstruction, this may make the initial hyper-rho resonance difficult to establish. To better find it we consider the pair production of hyper-rhos, leading to an eight-jet signal and consider this signal as a potential discovery in the near future. Before refurbishing, the LHC is running at a center of mass energy of 7 TeV. We present results for possible detection at this energy assuming an integrated luminosity of 1 fb^{-1}.

A Model with Colored Vector Bosons and Scalars.– As detailed in Ref. [2, 6, 7], we assume there is a new $SU(N_{\rm HC})$ gauge group, hypercolor, acting on a new set of fermions which also carry Standard Model color charges. By analogy with the spontaneous breaking of chiral symmetry in the Standard Model, one can derive an effective Lagrangian for new, massive color octets. These would be visible as a set of scalars, designated $\tilde{\pi}$, and a vector boson, $\tilde{\rho}$.

Their interactions are expressed by the following effective Lagrangian [7]:

$$\begin{aligned}\mathcal{L}_{\rm eff} \;=\; & -\frac{1}{4}G^{a}_{\mu\nu}G^{a\mu\nu} + \bar{q}i\not{D}q - \frac{1}{4}\tilde{\rho}^{a}_{\mu\nu}\tilde{\rho}^{a\mu\nu} + \frac{M_{\tilde{\rho}}^{2}}{2}\tilde{\rho}^{a}_{\mu}\tilde{\rho}^{a\mu} - g_{3}\epsilon\tilde{\rho}^{a}_{\mu}\bar{q}\gamma^{\mu}T^{a}q \\ & +\frac{1}{2}(D_{\mu}\tilde{\pi})^{a}(D^{\mu}\tilde{\pi})^{a} - M_{\tilde{\pi}}{}^{2}\tilde{\pi}^{a}\tilde{\pi}^{a} - ig_{\tilde{\rho}\tilde{\pi}\tilde{\pi}}f^{abc}\tilde{\rho}^{a\mu}(\tilde{\pi}^{b}D_{\mu}\tilde{\pi}^{c}) \\ & -\frac{3{g_{3}}^{2}\epsilon^{\mu\nu\rho\sigma}}{16\pi^{2}f_{\tilde{\pi}}}\,\mathrm{Tr}[\tilde{\pi}G_{\mu\nu}G_{\rho\sigma}] + i\chi g_{3}\,\mathrm{Tr}[G_{\mu\nu}[\tilde{\rho}^{\mu},\tilde{\rho}^{\nu}]]\,. \end{aligned} \tag{1}$$

The number of hypercolors has been set to be $N_{\rm HC} = 3$ for simplicity. $G_{\mu,\nu}$ and q are Standard Model (SM) gluon and quark fields, while a is a color index. Under the assumptions of the model, Kilic et al. have derived most of the parameters in terms of a single variable, $m_{\tilde{\rho}}$. They set $\epsilon \simeq 0.2$, $g_{\tilde{\rho}\tilde{\pi}\tilde{\pi}} \simeq 6$, $M_{\tilde{\pi}} \simeq 0.3 \times M_{\tilde{\rho}}$, and

$$f_{\tilde{\pi}} \simeq f_{\pi} \times \frac{M_{\tilde{\rho}}}{m_{\rho}}\,. \tag{2}$$

We use exactly these relations so a value of $m_{\tilde{\pi}}$ determines the value of $M_{\tilde{\rho}}$. The last term of the Lagrangian in Eq. (1) contains a free parameter χ which cannot be extrapolated from the Standard Model.

Production Cross Section.– In our model the colorons, $\tilde{\rho}$, can only be pair produced at the LHC via gluon fusion. (Single production of a coloron resonance is suppressed due to the small $q\bar{q}\tilde{\rho}$ coupling, as well as small quark-antiquark luminosity at the LHC). We calculate the cross section at the early LHC for $pp \to \tilde{\rho}\tilde{\rho} \to 4\tilde{\pi} \to 8g + X$ with the parton distribution functions of CTEQ6L1 [8]. The factorization scale as well as the renormalization scale is chosen to be (a) the coloron mass ($M_{\tilde{\rho}}$) for the coloron signal and (b) the root mean square transverse momentum ($\sqrt{\langle p_T^2\rangle}$) of all eight jets for the physics background, with the leading order evolution of the strong coupling. For simplicity, the K factor is taken to be one for both the signal and the background.

We have evaluated the cross section for $pp \to \tilde{\rho}\tilde{\rho} \to 4\tilde{\pi} \to 8g + X$ from gluon fusion and quark-antiquark fusion in two ways, (a) with complete matrix element involving Breit-Wigner resonances of both $\tilde{\rho}$ and $\tilde{\pi}$, and (b) with matrix elements involving Breit-Wigner resonance of $\tilde{\rho}$ for $pp \to \tilde{\rho}\tilde{\rho} \to 4\tilde{\pi} + X$ and narrow width approximation (NWA) for $\tilde{\pi} \to gg$. A new model has been added in MadGraph [9, 10] with new interactions and new particles to generate matrix elements squared for all processes in (a) and (b). In the narrow width approximation, the cross section for $pp \to \tilde{\rho}\tilde{\rho} \to 4\tilde{\pi} \to 8g + X$ can be thought of as the production cross section $\sigma(pp \to \tilde{\rho}\tilde{\rho} \to 4\tilde{\pi} + X)$ multiplied by the branching fraction of hyper-pions decay into gluon pairs $B(\tilde{\pi} \to gg) = 1$. In addition, we have checked $|M|^2(gg \to \tilde{\rho}\tilde{\rho} \to 4\tilde{\pi})$ analytically. The numerical output from MadGraph gives excellent agreement with that from our analytic expressions.

With energy-momentum smearing, the cross section in the narrow width approximation (NWA) agrees very well with that evaluated via a Breit-Wigner resonance (BWR) for most parameters that we have chosen. The ATLAS detector specifications [11] have been adopted to model these effects by Gaussian smearing the momenta of the jets,

$$\frac{\Delta E}{E} = \frac{0.60}{\sqrt{E}} \oplus 0.03\,, \tag{3}$$

with individual terms added in quadrature.

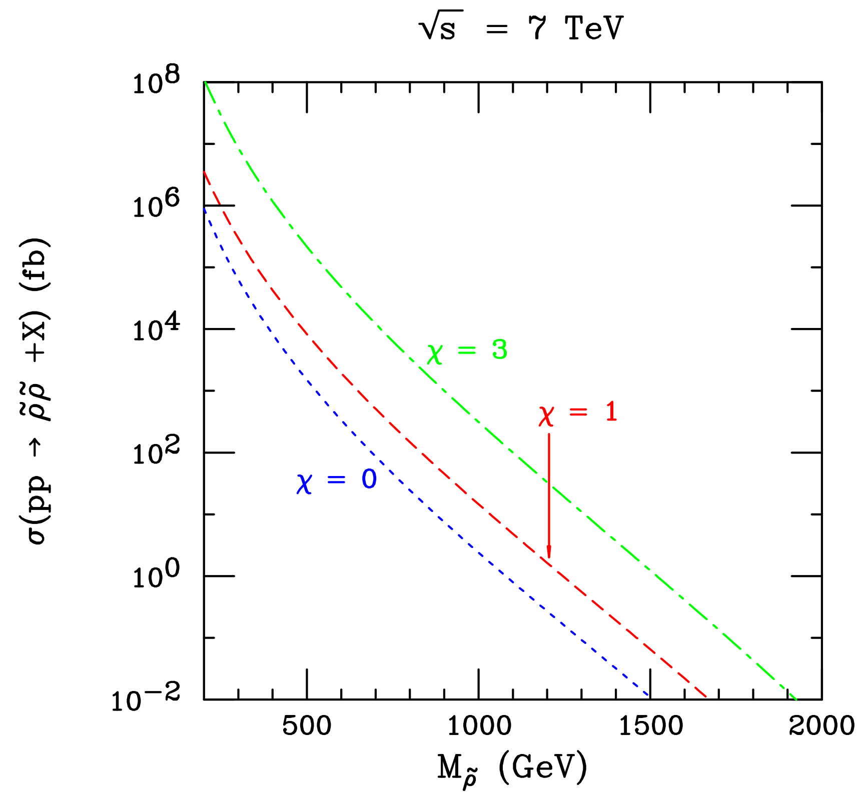

FIG. 1: The cross section of $pp \to \tilde\rho\tilde\rho + X$ for $\chi = 1, 0$ and 3 at the LHC with $\sqrt{s} = 7$ TeV, as a function of $M_{\tilde\rho}$.

To demonstrate that colorons can be produced copiously at the early LHC, we show, in Fig. 1, the cross section for $pp \to \tilde\rho\tilde\rho + X$ for a few values of χ. For this figure we took the scale to be $Q = M_{\tilde\rho}$, and did not make any cuts. For $gg \to \tilde\rho\tilde\rho$ we use an analytic expression for the square of the matrix element, summed over polarizations,

$$
\begin{aligned}
\sum_{\rm pol} |T|^2 \;=\;& \frac{Y^2(1-z^2)^2}{(1-\beta^2 z^2)^2}\frac{E^4}{M_{\tilde\rho}^4}\left[12 - 12Y + (5+z^2)Y^2\right] \\
&+ \frac{Y^2(1-z^2)}{(1-\beta^2 z^2)^2}\frac{E^2}{M_{\tilde\rho}^2}\left[16(1+3z^2) - 2(11+18z^2)Y + (5+9z^2+3z^4)Y^2\right] \\
&+ \frac{1}{(1-\beta^2 z^2)^2}\Bigg[8\left(16 + 3\frac{M_{\tilde\rho}^4}{E^4}\right) - 256Y + (160+16z^2+36z^4)Y^2 \\
&\qquad\qquad - (32+22z^2+24z^4)Y^3 + (2+5z^2+4z^4+2z^6)Y^4\Bigg] \\
&+ \frac{1}{1-\beta^2 z^2}\left[-6\left(16 + 4\frac{M_{\tilde\rho}^2}{E^2} + \frac{M_{\tilde\rho}^4}{E^4}\right) + 140Y - (58+24z^2)Y^2 + 3(1+4z^2)Y^3 - z^4Y^4\right] \\
&+ 28 + 6\frac{M_{\tilde\rho}^2}{E^2} - 3(1-\beta^2 z^2) - 16Y + 4Y^2 \qquad (4)
\end{aligned}
$$

where E is the gluon energy, z is the cosine of the scattering angle, $\beta^2 = 1 - M_{\tilde\rho}^2/E^2$, and $Y = 1 - \chi$. Clearly the theory is only unitary for $\chi = 1$ where the terms which

grow with energy are absent; in the remainder of the paper that is the only value we will use. For $\chi = 1$ we have checked that our results for $gg \to \tilde{\rho}\tilde{\rho}$ are consistent with those of Refs. [2, 3, 7] for $\sqrt{s} = 14$ TeV.

Physics Background.– We compute the cross section of the dominant eight jet physics background with the matrix-element generator COMIX [12], interfaced with the event generator SHERPA [13]. COMIX adopts color-dressed Berends-Giele recursion relations [14, 15] to construct QCD amplitudes. The physics backgrounds included are, in order of importance, $gq \to 7g1q$, $gg \to 8g$, $qq \to 6g2q$, and $gq \to 5g2q1\bar{q}$. For $M_{\tilde{\pi}} \gtrsim 230$ GeV, the $6g2q$ process becomes larger than the $8g$ process.

MadGraph employs traditional Feynman diagrams. It can calculate matrix elements with at most five outgoing gluons from gluon fusion. We have compared the cross section of $gg \to 4g$ with MadGraph and COMIX and have found excellent agreement.

To reduce the large QCD physics background, we require that in each event there should be eight jets ($j = g, q, \bar{q}$) with lower limits on their transverse momenta of

$$p_T(j_1, \cdots, j_8) \geq 250, 200, 160, 120, 80, 60, 40, 20 \text{ GeV}\,, \tag{5}$$

a pseudo-rapidity for each jet of $|\eta(j)| < 2.5$, and angular separation for each pair of jets $\Delta R = \sqrt{\Delta\phi^2 + \Delta\eta^2} > 0.5$.

Discovery Potential at the Early LHC.– To study the discovery potential of $pp \to \tilde{\rho}\tilde{\rho} \to \tilde{\pi}\tilde{\pi} \to 8g+X$, we evaluate cross sections for the SM backgrounds as described in the previous section. In addition, we have considered two types of mass cuts: (i) relative mass cuts and (ii) fixed mass cuts.

The relative mass cut requires that within each event there must be eight jets, which can be arranged into four pairs of jets that have invariant mass within ΔM_{2j} of one another, and there must be distinct pairs of four jets that have invariant mass within ΔM_{4j} of each other. We have chosen (a) $\Delta M_{2j} \leq 30$ GeV and $\Delta M_{4j} \leq 60$ GeV or (b) $\Delta M_{2j} \leq 50$ GeV and $\Delta M_{4j} \leq 100$ GeV.

The fixed mass cut requires that within each event, there must be eight jets with four pairs of jets that have invariant mass within a bin of width $\pm\Delta M_{2j}$ centered at $M_{\tilde{\pi}}$, and there must be two groups of four jets that have invariant mass within a bin of width $\pm\Delta M_{4j}$ centered at $M_{\tilde{\rho}}$. We have chosen (a) $|M_{2j} - M_{\tilde{\pi}}| \leq 0.10 M_{\tilde{\pi}}$ and $|M_{4j} - M_{\tilde{\rho}}| \leq 0.15 M_{\tilde{\rho}}$ or (b) $|M_{2j} - M_{\tilde{\pi}}| \leq 0.15 M_{\tilde{\pi}}$ and $|M_{4j} - M_{\tilde{\rho}}| \leq 0.20 M_{\tilde{\rho}}$.

We define the signal to be observable if the lower limit on the signal plus background is larger than the corresponding upper limit on the background [16], namely,

$$L(\sigma_s + \sigma_b) - N\sqrt{L(\sigma_s + \sigma_b)} > L\sigma_b + N\sqrt{L\sigma_b} \tag{6}$$

which corresponds to

$$\sigma_s > \frac{N^2}{L}\left[1 + 2\sqrt{L\sigma_b}/N\right]\,. \tag{7}$$

Here L is the integrated luminosity assumed to be 1 fb^{-1}, σ_s is the cross section of the coloron signal, and σ_b is the background cross section. The parameter N specifies the level or probability of discovery. We take $N = 2.5$, which corresponds to a 5σ signal.

To assess the discovery potential we present in Fig. 2 the cross sections of the coloron signal, and the physics background, after acceptance cuts and relative mass cuts, versus

$M_{\tilde{\pi}}$. Also shown are the background cross section for the SM processes with the relative mass cuts discussed above. In addition, we present the minimal signal cross section that is required to establish a 5σ signal with relative mass cuts (a) or (b) as given above.

We note that a narrower relative mass cut (a) has the potential to discover the colorons and hyper-pions up to $M_{\tilde{\pi}} = 200$ GeV ($M_{\tilde{\rho}} = 667$ GeV). A wider relative mass cut (b) will allow more background events, and thus has a slightly reduced discovery reach of $M_{\tilde{\pi}} = 180$ GeV ($M_{\tilde{\rho}} = 600$ GeV). In addition, relative mass cut (a) can improve the signal to background ratio (σ_s/σ_b) significantly.

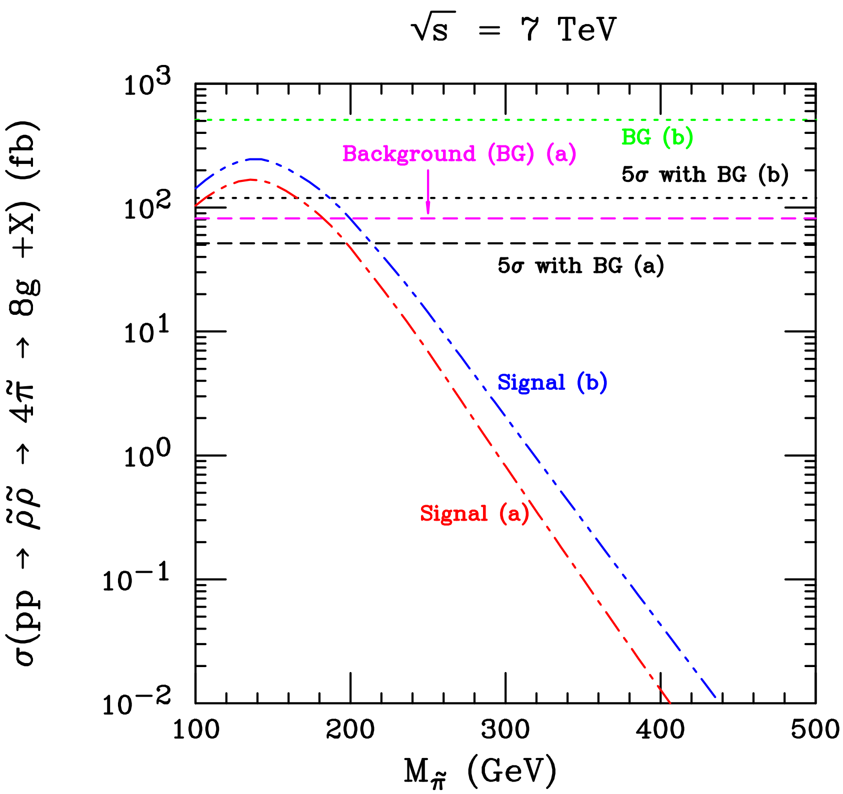

FIG. 2: The cross section for $pp \to \tilde{\rho}\tilde{\rho} \to 4\tilde{\pi} \to 8g + X$ at the LHC with $\sqrt{s} = 7$ TeV, as a function of $M_{\tilde{\pi}}$. We have applied kinematic cuts on p_T, η, ΔR and two sets of relative mass cuts: (a) $\Delta M_{2j} < 30$ GeV and $\Delta M_{4j} < 60$ GeV [red, dot-dash], or (b) $\Delta M_{2j} < 50$ GeV and $\Delta M_{4j} < 100$ GeV [blue, dot-dot-dash]. Also shown are the background cross section for the SM processes from the production of $8g$, $7g1q$, $6g2q$, and $5g3q$ with relative mass cut (a) [magenta, dash] and relative mass cut (b) [green, dot]. In addition, we present the minimal signal cross section that is required by a 5 sigma criterion with relative mass cut (a) [dash] and relative mass cut (b) [dot].

Figure 3 shows cross sections of the coloron signal (σ_s) and the physics background (σ_b) from the production of $8g$, $7g1q$, $6g2q$, and $5g3q$, with acceptance cuts and fixed mass cuts versus $M_{\tilde{\pi}}$. We have replaced the relative mass cuts of Fig. 2 with two sets of fixed mass cuts: (a) $|M_{2j} - M_{\tilde{\pi}}| < 0.10 M_{\tilde{\pi}}$ and $|M_{4j} - M_{\tilde{\rho}}| < 0.15 M_{\tilde{\rho}}$, or (b) $|M_{2j} - M_{\tilde{\pi}}| < 0.15 M_{\tilde{\pi}}$ and $|M_{4j} - M_{\tilde{\rho}}| < 0.20 M_{\tilde{\rho}}$. Also shown is the minimal signal cross section that is required by a 5 sigma criterion. If the background has fewer than 16 events assuming 1 fb^{-1} of luminosity, we employ the Poisson distribution and require that the Poisson probability for the SM background to fluctuate to this level should be less than 2.85×10^{-7}. For $M_{\tilde{\pi}} = 100$ GeV it

is very time consuming to get a convergent cross section for σ_b. To improve the stability we have applied somewhat less stringent p_T cuts than those given in Eq. (5),

$$p_T(j_1, \cdots, j_8) \geq 200, 150, 120, 100, 80, 60, 40, 20 \text{ GeV} \tag{8}$$

respectively. Therefore, the background cross section (red, cross) and the corresponding 5σ signal cross section (green, diamond) for $M_{\tilde{\pi}} = 100$ GeV are presented with symbols. We note that even with lower p_T cuts, the background is negligible.

If the ATLAS [11] and CMS [17] detectors have excellent mass resolution, we will be able to apply the narrower fixed mass cut (a), which has the potential to discover the colorons and hyper-pions up to $M_{\tilde{\pi}} = 220$ GeV ($M_{\tilde{\rho}} = 733$ GeV). A wider fixed mass cut (b) will allow more background events, which results in a slightly reduced discovery reach of $M_{\tilde{\pi}} = 210$ GeV ($M_{\tilde{\rho}} = 700$ GeV).

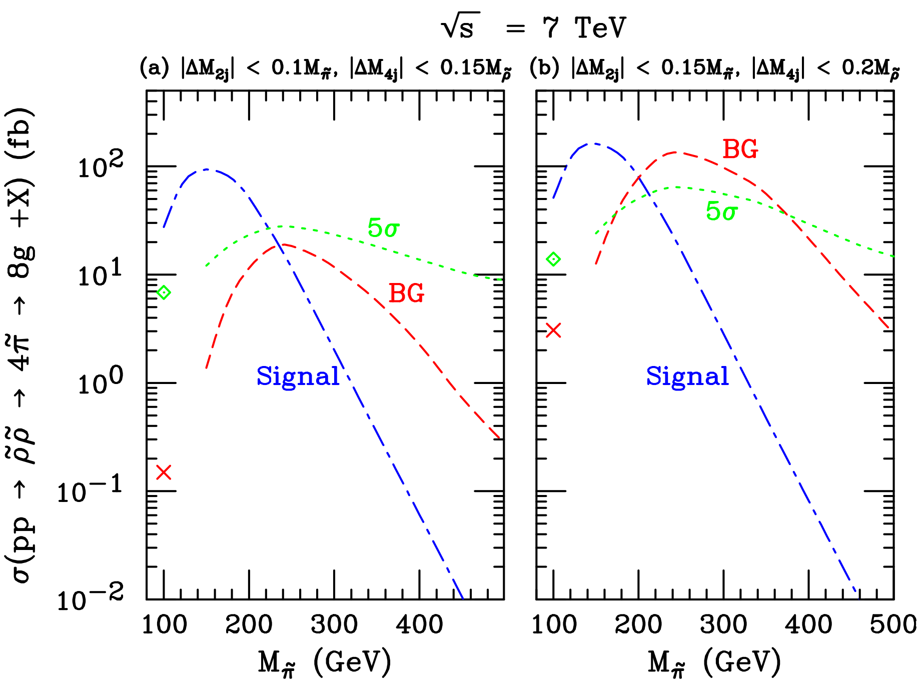

FIG. 3: The cross section for $pp \to \tilde{\rho}\tilde{\rho} \to 4\tilde{\pi} \to 8g + X$ (blue, dot-dash) at the LHC with $\sqrt{s} = 7$ TeV, as a function of $M_{\tilde{\pi}}$ with acceptance cuts on p_T, η, and ΔR. We have applied two sets of fixed mass cuts: (a) $|M_{2j} - M_{\tilde{\pi}}| < 0.10 M_{\tilde{\pi}}$ and $|M_{4j} - M_{\tilde{\rho}}| < 0.15 M_{\tilde{\rho}}$, or (b) $|M_{2j} - M_{\tilde{\pi}}| < 0.15 M_{\tilde{\pi}}$ and $|M_{4j} - M_{\tilde{\rho}}| < 0.20 M_{\tilde{\rho}}$. Also shown are the SM background cross section (σ_b)(red, dash) and the minimal signal cross section that is required by a 5 sigma criterion (green, dot). For $M_{\tilde{\pi}} = 100$ GeV, the 5σ signal cross section and σ_b (green diamond, red cross) are calculated with lower p_T cuts.

Conclusions.– We have demonstrated that colorons and hyper-pions can be produced abundantly at the early stage of the LHC with a center of mass energy $\sqrt{s} = 7$ TeV and an integrated luminosity of 1 fb^{-1}. With realistic acceptance cuts as well as relative mass cuts or fixed mass cuts, the physics background can be significantly reduced to establish a 5σ signal for $M_{\tilde{\pi}} \lesssim 220$ GeV ($M_{\tilde{\rho}} \lesssim 733$ GeV).

If the center of mass energy can be raised or the integrated luminosity increased the discovery potential of colorons at the early LHC will be significantly improved. The discovery

potential at the LHC for colorons and hyper-pions will be greatly enhanced with the full center of mass energy $\sqrt{s} = 14$ TeV and integrated luminosity $L = 30 - 300$ fb^{-1} [18].

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